paper no.

56-A--130

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

29 WEST 39TH STREET • NEW YORK 18, N. Y.

ESTIMATION OF TEMPERATURE PATTERNS IN MULTIPLY-SHIELDED SYSTEMS

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Contributed by the Heat Transfer Division for presentation at the ASME Annual Meeting, New York, N. Y - November 25-30,1956. (Manuscript received at ASME Headquarters July 31, 1956)

Written discussion on this paper will be accepted up to January 10,1957.

(Copies will be available until October 1, 1957)

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ABSTRACT

The method of difference equations is applied to the problem of estimating temperature patterns in multiply-shielded systems. Results are obtained in terms of elementary functions which can be readily evaluated. The equations are employed to examine the effect of shield curvature, end radiation and gas radiation on the temperature recorded by a shielded thermocouple for a variety of operating conditions. Calculations indicate that whereas shield curvature and gas radiation may be neglected in a well-designed shielded thermocouple, end radiation can result in a substantial error in the thermocouple-temperature measurement.

INTRODUCTION

A schematic diagram of a multiply-shielded system is shown in Fig. 1. The shield system is assumed to be located in a flowing gas stream of temperature T_g which is, in turn, contained in a duct with a wall temperature T_w.

The temperature of the central body T_1, in the absence of any shielding, may be computed by solving a heat-balance equation which considers radiant heat exchange with the duct wall, convection and radiation to and/or from the gas stream and any heat generated or absorbed by the shield. Conduction is neglected since this effect can usually be made negligible by suitable insulation of the supports. An additional heat balance equation is written for each added shield. The system of equations describing the shield array shown in Fig. 1 contains η unknown temperatures among the η algebraic relations. This system of equations may be solved in a number of ways.

The use of difference equations in the analysis of multiply-shielded systems appears to have been neglected in the engineering literature. This mathematical procedure, however, yields equations in terms of elementary functions which are readily evaluated. This paper is, therefore, primarily intended to present an engineering method for rapidly estimating the temperatures in configurations with thermal-radiation shields.

ASSUMPTIONS

The following assumptions are made:

1 The convection coefficient h_c is constant within the shield system.

2 The linearized radiation coefficient h_r is constant within the shield system.

3 The stream temperature T_g is uniform.

4 The wall temperature T_w of the duct enclosing the shield system is uniform.

5 The temperature of each shield T_k is uniform.

6 Conduction losses are negligible.

The linearized radiation coefficient h_r is generally computed as:

$$h_r = 4\sigma\epsilon\bar{T}^3$$

where $\bar{T}$ is an "average" temperature in the shield system. The calculation of the convective heat transfer coefficient h_c and the "effective" gas stream temperature Tg has been amply discussed in many references (1),[1] (5) and is not included here.

[1] Underlined numbers in parentheses refer to the Bibliography at the end of the paper.

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By J. G. Bartas and E. Mayer

NOMENCLATURE

The following nomenclature is used in this paper:

A,B,C,D = Constants

E = end radiation factor

F = intra-shield radiation factor

G = gas-shield radiation factor

h = heat transfer coefficient, (Btu/hr-ft^2-deg R)

k = integer (1, 2, ...n)

n = number of probe elements

T = temperature, Deg R.

ΔT = $T_g - T$, deg R.

ϵ = emissivity

σ = Boltzmann constant, (Btu/hr-ft^2-deg R^4)

Subscripts

c = convection

g = gas

i = innermost

o = outermost

r = radiation

w = wall

1,2,...k, ...n - index refers to shield element

Other symbols are defined as they are introduced in the text.

A SIMPLE SHIELD SYSTEM

The following analysis of a simple shield system illustrates the general analytical procedure and, furthermore, leads to results closely approximating those of more complex systems described in the next section.

Consider a shield system consisting of a set of plane parallel shields arranged about a central body as shown in Fig. 1. Assume that the external radiation and convection heat-transfer coefficients are equal to the internal radiation and convection heat-transfer coefficients, respectively. Also, assume that end radiation to and from the shield assembly is negligible and there is no interchange of radiation between the shield system and the gas stream.

With reference to Fig. 1 the preceding assumptions lead to the following heat balance equation, per unit shield surface area, for the shield element with index k

$$2h_c(T_g - T_k) + h_r(T_{k-1} - T_k) + h_r(T_{k+1} - T_k) = 0 \quad 1 < k < n \quad . \quad . \; (1)$$

It is convenient to employ the temperature difference identity.

$$\Delta T \equiv T_g - T$$

in terms of which the heat-balance equation becomes

$$\Delta T_{1+1} - 2A \Delta T_k + \Delta T_{k-1} = 0 \quad 1 < k < n \quad . \quad . \quad . \quad . \quad . (2)$$

where

$$A = \frac{h_c + h_r}{h_r}$$

Equation (2) is a homogeneous linear difference equation of the second order with constant coefficients. The solutions of difference equations of this type, where A is a real constant, are well known. For the case where $A > 1$ the general solution is of the form (reference 2 page 241.)

$$\Delta T_k = c_1 e^{\alpha k} + c_2 e^{-\alpha k} \quad . \quad . \quad . \quad . \; (3)$$

where α is defined by cosh = A, and C_1 and C_2 are constants which are determined by the boundary conditions.

The boundary conditions are given by the heat-balance equations written for the central body (k = 1) and the outermost shield (k = n). These equations are

$$h_c(T_g - T_k) + h_r(T_{k+1} - T_k) = 0 \qquad k = 1 \quad . \quad . \quad . \quad (4)$$

$$2h_c(T_g - T_k) + h_r(T_{k-1} - T_k) + h_r(T_{k+1} - T_k) = 0 \qquad k = n \quad . \quad . \quad . \quad (5)$$

In terms of the temperature difference ΔT the boundary conditions become

$$A\Delta T_1 - \Delta T_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$2A\Delta T_n - \Delta T_{n-1} = \Delta T_w \quad . \quad . \quad . \quad . \quad (7)$$

The constants C_1 and C_2 are determined by substitution of Equation (3) into the boundary conditions, Equations (6) and (7), which yield

$$C_1 = \frac{e^{-\alpha}}{e^{\alpha h} + e^{-\alpha n}} \Delta T_w \quad . \quad . \quad . \quad . (8)$$

$$C_2 = \frac{e^{\alpha}}{e^{\alpha n} + e^{-\alpha n}} \Delta T_w \quad . \quad . \quad . \quad . (9)$$

Inserting these results into Equation (3) and simplifying, we arrive at a simple expression for the temperature of any element in the simple shield system:

$$\frac{\Delta T_k}{\Delta T_w} = \frac{\cosh(k-1)\alpha}{\cosh n} \qquad k = 1,2,\ldots n \quad . \quad (10)$$

The temperature difference at the central body $(T_g - T_1)$ may be readily computed from the relation obtained by setting k = 1 in Equation (10)

$$\frac{T_g - T_1}{T_g - T_w} = \frac{1}{\cosh n\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Equation (11) is convenient for estimating the number of radiation shields necessary to suppress lateral radiant heat transfer in a multiply-shielded system to a desired level. Equation (11) is plotted in Fig. 2, where curves of $\Delta T_1/\Delta T_w$ versus n for h_c/h_r values of $\frac{1}{2}$, 1 and 2 are shown. Note that Equation (11) is valid for gas temperatures either greater or less than the wall temperature.

THE GENERAL SOLUTION

The method of determining the temperature pattern in a shield system subject to more general boundary conditions is analogous to the method outlined in the foregoing. A schematic diagram of a circular cylindrical multiple-shielded system is shown in Fig. 3. The heat-balance equation for the k^{th} element, including end radiation effects, is

$$(C h_c + G h_r)(T_g - T_k) + F h_r(T_{k-1} - T_k) + h_r(T_{k+1} - T_k) + E h_r(T_w - T_k) = 0$$

$$1 < k < n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The parameter G is a measure of the amount of net radiation between the gas and the k^{th} shield in relation to the lateral radiation between the shields. It includes end radiation to and from the gas, as well as any radiant heat exchange with the gas flowing between the shields.

The radiant-interchange configuration factor F is the fraction of radiant energy leaving the surface of the k^{th} element, which is intercepted by the surface of the $(k-1)^{th}$ element. For circular cylindrical shields

$$F = \frac{R_{k-1}}{R_k}$$

where R is the shield radius (4).

The factor E is a measure of the amount of end radiation between the wall and the k^{th} element. Values for E may also be computed with the aid of reference (4).

The parameters E, F, and G are assumed constant for all internal elements of the shield assembly. If they are considered as functions of the index k, the coefficients in the difference equation are variables, and no general solution is usually obtainable.

Equation (12) is written in the temperature-difference notation as:

$$\Delta T_{k+1} - \frac{2h_c + (1 + E + F + G)h_r}{h_r} \Delta T_k + F\Delta T_{k-1} = E\Delta T_W \quad 1 < k < n \quad . \quad . \quad .(13)$$

This equation is reduced to a "standard" form similar to Equation (2) by the use of

$$\Delta T_k = F^{\frac{k}{2}} \Delta U_k \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Combining Equations (13) and (14) we arrive at

$$\Delta U_{k+1} - 2B\Delta U_k + \Delta U_{k-1} = \frac{E}{F^{\frac{k+1}{2}}} \Delta T_W \quad . \quad . \quad .(15)$$

where

$$B = \frac{2h_c + (1 + E + F + G)h_r}{2h_r F^{\frac{1}{2}}}$$

Equation (15) is a nonhomogeneous linear equation of the second order with constant coefficients.

The general solution of a nonhomogeneous equation of the form of Equation (15) may be expressed as the sum

$$\Delta U_k = \Delta U_k^{(H)} + \Delta U_k^{(P)} \quad . \quad . \quad . \quad . \quad (16)$$

where $\Delta U_k^{(H)}$ is the general solution of the homogeneous equation

$$\Delta U_{k+1} - 2B\Delta U_k + \Delta U_{k-1} = 0$$

and $\Delta U_k^{(P)}$ is any particular solution of Equation (15).

The general solution of the homogeneous equation is [cf. Equations (2) and (3)]

$$\Delta U_k^{(H)} = D_1 e^{\beta k} + D_2 e^{-\beta k} \quad . \quad . \quad . \quad (17)$$

where

$$B = \cosh \beta$$

The particular solution can be derived by the method of undetermined coefficients. It may be verified that a particular solution is

$$\Delta U_k^{(P)} = \frac{-E \Delta T_W}{F^{\frac{k}{2}}(2BF^{\frac{1}{2}}-1-F)} \quad . \quad . \quad . \quad . \quad . (18)$$

by substituting this expression in Equation (15).

The general equation for ΔT_k is therefore

$$\Delta T_k = F^{\frac{k}{2}}\left[D_1 e^{\beta k} + D_2 e^{-\beta k}\right] - \frac{E \Delta T_W}{2BF^{\frac{1}{2}}-1-F} \quad 1 < k < n \quad . \quad . \quad . \quad (19)$$

The two boundary conditions for the shield system shown in Fig. 3, including end radiation effects, are

$$(h_c + h_{ci} + G_i h_r)(T_g - T_k) + h_r(T_{k+1} - T_k) + E_i h_r(T_W - T_k) = 0 \quad k = 1 \quad . \quad . \quad . \quad (20)$$

$$(h_c + h_{co} + G_o h_r)(T_g - T_k) + F h_r(T_{k-1} - T_k) + h_{ro}(T_W - T_k)$$

$$+ E_o h_{ro}(T_W - T_k) = 0 \quad k = n \quad . \quad . \quad . \quad . \quad . \quad . (21)$$

In the temperature-difference notation the boundary conditions are written as

$$J_1 F^{\frac{1}{2}} \Delta T_1 - \Delta T_2 = E_i \Delta T_W \quad . \quad . \quad . \quad . \quad (22)$$

$$J_n F^{\frac{1}{2}} \Delta T_n - F \Delta T_{n-1} = (1 + E_o) \frac{h_{ro}}{h_r} \Delta T_w \quad . \quad . \quad . \quad (23)$$

where

$$J_1 = \frac{h_c + h_{ci} + (1 + E_i + G_i) h_r}{F^{\frac{1}{2}} h_r}$$

$$J_n = \frac{h_c + h_{co} + (F + G_o) h_r + (1 + E\ ) h_{ro}}{F^{\frac{1}{2}} h_r}$$

The constants D_1 and D_2 in Equation (19) are evaluated with the aid of the boundary condition Equations (22) and (23). After some algebraic manipulation the final equation for $\frac{\Delta T_k}{\Delta T_w}$ is

$$\frac{\Delta T_k}{\Delta T_w} = F^{\frac{k}{2}} \left\{ \frac{\phi_1 \psi_1 + \phi_2 \psi_2}{(J_1 J_n - 1) \sinh (n-1) \beta \ - (J_1 + J_n - 2B) \sinh (n-2) \beta} \right\} + \\ - \frac{E}{2BF^{\frac{1}{2}} - 1 - F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where

$$\psi_1 = J_n \sinh (n-k) \beta - \sinh (n-k-1) \beta$$

$$\psi_2 = J_1 \sinh (k-1) \beta - \sinh (k-2) \beta$$

$$\phi_1 = \frac{E_i}{F} + \frac{(J_1 F^{\frac{1}{2}} - 1) E}{F92BF^{\frac{1}{2}} - 1 - F)}$$

$$\phi_2 = \frac{(1 + E_o) h_{ro}}{F^{\frac{n+1}{2}} h_r} + \frac{(J_n F^{\frac{1}{2}} - F) E}{F^{\frac{n+1}{2}} (2BF^{\frac{1}{2}} - 1 - F)}$$

The refinements of Equation (24) are inconsistent with the crude initial assumptions. However, the equation may be used to determine the relative importance of several parameters which have been assumed to influence the temperature pattern of the shield system.

EFFECT OF SHIELD CURVATURE IN THERMOCOUPLE APPLICATIONS

The first of these parameters to be considered is shield curvature. The ratio $\frac{\Delta T_1}{\Delta T_w}$ is plotted as a function of the number of probe elements in Fig. 4 for several values of h_c/h_r with the configuration factor F taken as 1.0, 0.9 and 0.8. The innermost probe element (k=1) has been selected for comparison since this element is of most interest in shielded thermocouple applications where curvature effects may be important.

It is seen in Fig. 4 that the influence of sharply-curved and/or widely spaced curved shields on the temperature of the innermost element of a shielded thermocouple system is secondary in relation to the influence of the number of shields and to variations in convective and radiant heat transfer. Furthermore, the factor F is close to unity for typical shielded thermocouple geometries (.). These assemblies may therefore be analyzed as parallel shield assemblies with only higher order errors incurred due to neglecting the effect of shield curvature.

END RADIATION

A shielded thermocouple is also a convenient model to use in estimating the contribution of end radiation to variations in the temperature patterns of a shield system. The ratio $\Delta T_1/\Delta T_w$ is plotted against the number of thermocouple elements η in Fig. 5 for end radiation coefficient (E) values of 0.00, 0.01 and 0.02. The end radiation factor E is of the order of 0.01 to 0.02 for typical thermocouple shield assemblies (3).

It can be seen in Fig. 5 that end radiation becomes the dominating influence as the number of shields increases. This is due to the fact that end radiation depends very little on the number of shields and becomes completely independent of the number of shields once lateral radiation is suppressed. For example, if $h_c/h_r = 20$ and E = 0.02, the ratio $\Delta T_1/\Delta T_w$ is dependent on the number of shields, as follows:

Number of Shields (n-1)	$\frac{\Delta T_1}{\Delta T_w}$
1	.0390
2	.0094
3	.0043
4	.0034
5	.0033
∞	.00327

Fig. 5 quantitatively illustrates the well-known fact that end radiation should not be ignored if a high degree of accuracy in thermocouple temperature measurements is required.

GAS RADIATION

It is not feasible to plot the effect of varying G for a thermocouple since the curves for various values of G tend to overlap. This effect is illustrated by the following tabulated results:

$$\frac{\Delta T_1}{\Delta T_w}$$

Number of Shields (n-1)	Value of G 0.00	0.01
1	.0948	.0941
2	.0271	.0269
3	.0089	.0089
4	.00406	.00406

Conditions - $h_c = h_{ci} = h_{co} = h_r = h_{ro}$

$$E = E_i = E_o = 0.01$$

$$F = 1.0$$

As the number of shields increases the temperature of the probe approaches the gas temperature and no appreciable radiant energy is exchanged between the thermocouple and the gas stream.

CONCLUSIONS

The preceding analysis presents an engineering method for rapidly estimating the temperatures in a system which contains an array of thermal radiation shields. The application of the calculus of finite differences leads to an equation which may be used to compute the temperature of <u>any</u> shield once the system parameters have been evaluated. The method applied to the problem of temperature measurement by shielded thermocouples produces results which are qualitatively well known, e.g. shield curvature and gas radiation effects are secondary quantities in a well-shielded thermocouple; end radiation may be important. The quantitative influence of convection, radiation, the number of shields, etc. on the thermocouple reading may be seen readily from the graphs which are included.

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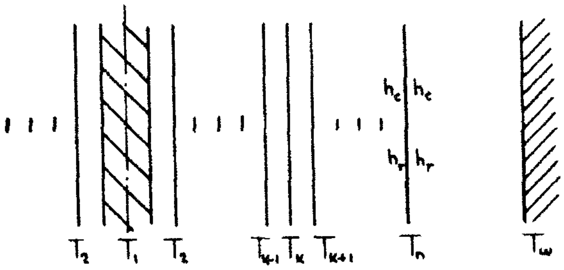

Fig.1 Schematic diagram of a simple shield system

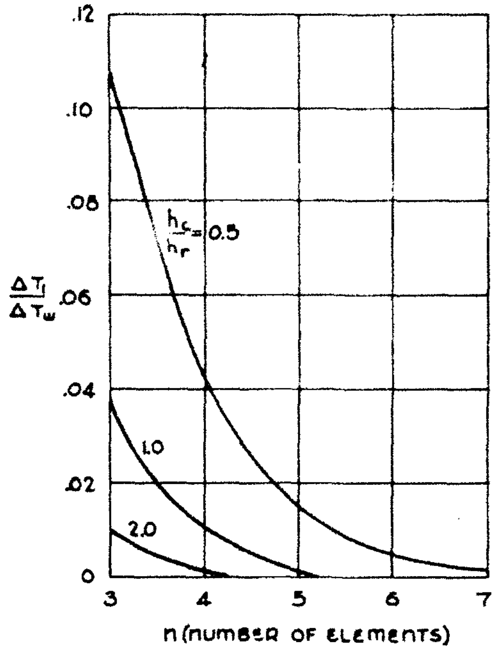

Fig.2 $\Delta T_1/\Delta T_w$ versus n for h_c/h_r ratios of 0.5, 1.0, and 2.0

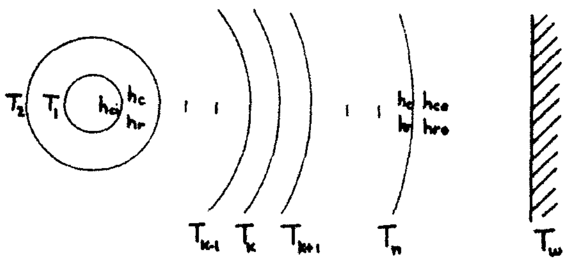

Fig.3 Schematic diagram of a circular cylindrical shield system

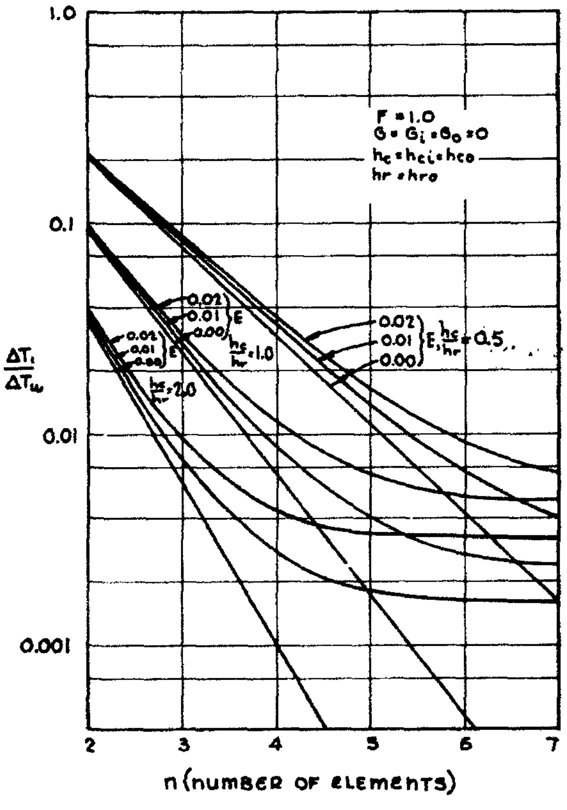

Fig.5 Effect of end radiation factor "E" on $\Delta T_1/\Delta T_w$

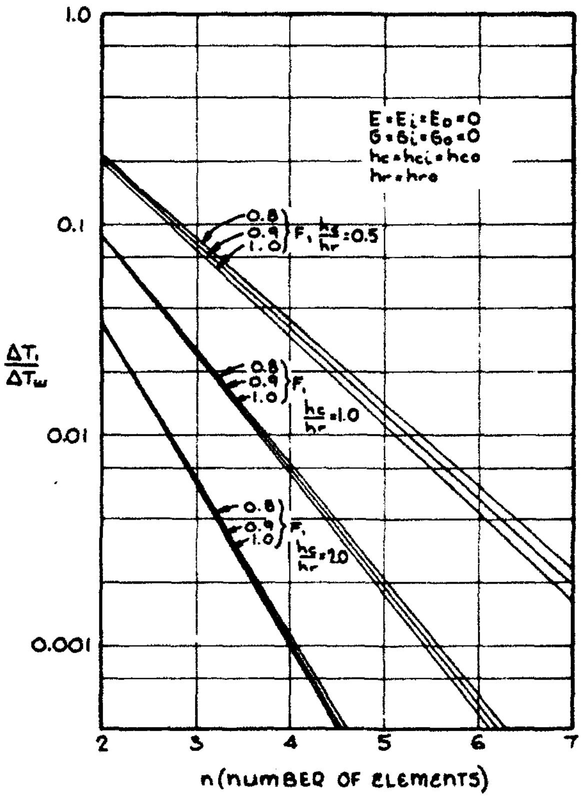

Fig.4 Effect of shield curvature factor "F" on $\Delta T_1/\Delta T_w$